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# *Theory and Mathematics in Building Dynamic Systems Models: What Prevails? A Reply to van der Maas and Raijmakers*

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In their commentary, van der Maas and Raijmakers (2000) made three main points that will be discussed in this paper. The first point concerns the criticisms that they raised against our connectionist model. We discuss their second point, criticisms of the logistic growth model, in conjunction with their phase transition model. Finally, we discuss the third issue of whether it is possible to draw general conclusions with regard to the behaviour of different dynamical systems models.

## THE CONNECTIONIST MODEL

van der Maas and Raijmakers (2000) criticised our connectionist model (Olthof *et al.*, 2000) for being unparsimonious and unsuitable for mathematical analysis. Our response to this point is threefold. First, given our aim of clarifying the empirical implications of a non-linear view of mother–child interaction, to us the issue of the parsimony of the to-be-used models was not as critical as van der Maas and Raijmakers considered it to be. For our purposes, it was sufficient that the connectionist model belongs to the class of non-linear models, with the additional advantage that it had been used in a developmental context before.

Second, we agree that connectionist models as a class generally are not parsimonious, in that it might well be possible to describe their behaviour by simpler mathematical models. It should be noted, however, that within the class of connectionist models, our model is not particularly unparsimonious. The architecture of the networks was based on what seemed to be minimally required to let them interact under conditions of varying external stress.

Third, even though we did not use the networks' capacity to learn in the simulation of interaction *per se*, the connectionist approach should not be dismissed too early because in future simulations it might be possible to let the interacting networks provide each others' learning environment, while simultaneously interacting with each other. This would be a way to simulate long-term developmental change as a result of the individual's interaction history.

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## THE LOGISTIC GROWTH AND PHASE TRANSITION MODELS

### *Differential versus difference equations*

We agree that the differential model has the advantage that time is seen as continuous rather than discrete. As was pointed out in footnote 1, we used difference equations for practical reasons only, i.e. to enable readers to easily reproduce the model with a spreadsheet programme. Moreover, our attempt to construct a differential model revealed that there were no major differences within the parameter ranges that were used in our simulations.

### *How input affects the child and the interpretation of $r$*

van der Maas and Raijmakers suggested making the input additive to the basic equation, and to define the  $r$  in the child's equation as arousal. Although this change definitely yields a simpler model, it also implies that rapid changes are possible in the carrying capacity, that is, in the maximum stable level that the child's behaviour can reach. This would mean that the highest level at which a particular child can continuously emit distress behaviour during a prolonged period of time varies enormously from one situation to the other, depending on the intensity of the external stressor, and on the extent to which the mother behaves responsively. As we know of no formal or anecdotal evidence indicating that such variations exist, this strikes us as implausible.

### *Conceptualizing caring behaviour*

We assumed caring to be an elementary type of behaviour that may have inborn components, and that is as dependent on arousal and emotions as is infants' distress. We further assumed that when interacting with the child, the mother continuously adjusts her behaviour to the child. Accordingly, we took the mother's caring behaviour to depend continuously and non-linearly on her individual characteristics and on the child's behaviour. In line with these assumptions, we modelled the mother's behaviour in a similar way as the child's behaviour, i.e. as a non-linear function.

van der Maas and Raijmakers, in contrast, modelled the mother's caring behaviour as a simple autonomously growing variable that switches on and off, depending on whether the child is in a negative mood. Although van der Maas and Raijmakers presented this modification as a correction of unnecessary complexities in our model, we consider this change to be a fundamental one because their model no longer represents the mother as an interacting complex system. This is not only at odds with our assumptions, but also with van der Maas and Raijmakers' own stated conviction that a mother and her child are 'two very complex systems'. A practical consequence of van der Maas and Raijmakers' simplification of caring behaviour is that it becomes difficult to directly compare the results of their simulations to those generated by our model.

Even though we consider our conceptualisation of caring behaviour to be more plausible than that of van der Maas and Raijmakers, it might nevertheless be the case that their simplification does not prevent the model from making empirically accurate predictions. Accordingly, we propose to treat the issue as an empirical question. Observing mother–infant dyads could help to specify the relationship between maternal behaviour and infants' distress. In our paper, we argued that dynamical systems models could help to determine the kind of

phenomena research should pay attention to. The question discussed here underlines this point in a slightly different way, in that trying to build a dynamical systems model makes clear what we do not know yet about the process we want to study. In that sense it sets an agenda for future research.

### *Slowing down auto-catalytic growth*

van der Maas and Raijmakers proposed to let the auto-catalytic growth of the child's distress slow down by a factor  $1-(D/K)^2$ , instead of by the simpler factor  $1-(D/K)$  that we used, the reason being that the resulting cubic model shows more interesting behaviour. This proposal strikes us as somewhat surprising, when seen in the light of van der Maas and Raijmakers' earlier plea for parsimony. In the interest of parsimony, we would prefer to start with the simplest possible factor, rather than with the more complex cubic model. However, as before, comparison with empirical data should help us to decide in favour of one of the models and such research might, of course, yield empirical evidence of sudden jumps that could lead one to prefer the cubic model (for a detailed example of this see Ruhland, 1998).

### *Patterns of distress*

van der Maas and Raijmakers' first two simulations differed from ours, in that the intensity of the external stressor was varied over time, whereas in our simulations stressor intensity and time were varied independently. This difference is a further hindrance to making a direct comparison between their simulations and ours. van der Maas and Raijmakers' third scenario is even less comparable to our simulations because the growth factor ( $r$ ) of the child's distress—interpreted as the child's arousal—was varied, whereas in our logistic growth model  $r$  depended on the mother's caring behaviour

### *General implications of using a dynamic systems approach*

The aim of our study was to clarify the empirical implications of applying a non-linear dynamic systems perspective to mother-child interaction. We hoped to provoke empirical researchers to collect the kind of data that are needed to evaluate claims about the non-linearity of interaction. Based on the results of two different models, we concluded that empiricists should set up their research in such a way that the data can reveal the existence of critical thresholds and paradoxical effects, both of which are likely to be unique for individual mother-child dyads. In terms of the design of studies, this would imply a focus on individual dyads, rather than on averaged group data, and on continuously measured interactive behaviour of mothers and children who have to adapt to varying external conditions.

van der Maas and Raijmakers argued that it makes little sense to look for general implications of the non-linear dynamic systems approach, on the grounds that even the relatively minor changes that they made to our logistic growth model produced quite different results. We do not consider this particular justification for their claim to be very convincing because, as was argued above, at least one of their changes to our model was not minor at all. Moreover, their simulations used varying stressor intensity, whereas ours did not. This is not to deny that van der Maas and Raijmakers are right when claiming that the specific results that are generated by different types of non-linear models can diverge widely, and that each model should be judged by its own merit.

Nevertheless, we think that the above summary of our results shows that there is a level of description at which it does make sense to identify general implications of applying a non-linear dynamic systems approach to mother–child interaction, some of these concerning the design of empirical studies. As soon as such studies have generated the relevant kind of data, we can start doing what van der Maas and Raijmakers recommend, i.e. judge each particular non-linear dynamic systems model by its own merit.

## CONCLUSION

The most important differences between our models and that of van der Maas and Raijmakers might well be a result of a difference in opinion about what should prevail when building a dynamic systems model. Our approach was to start with theoretical assumptions that were subsequently translated into models. As van der Maas and Raijmakers argued convincingly, the resulting models were too complex to allow mathematical analysis, which also deprived us of a way to demonstrate that the changes and effects that we loosely described as ‘rather abrupt’ and ‘paradoxical’ actually satisfied formal criteria for the use of such terms that have been specified in the literature.

When constructing their own model, van der Maas and Raijmakers seem to have worked the other way around, in that they adjusted one of our models in such a way that it can be analysed mathematically. However, in doing so, they violated a major assumption underlying our models, i.e. that in mother–child interaction *both* participants continuously adjust their behaviour to the other participant’s behaviour.

Accordingly, the question is: what should prevail, theoretical or model-technical demands? We agree with van der Maas and Raijmakers that mathematical analysis has many advantages and that models that can be analysed mathematically should—all other things being equal—be preferred over models that do not allow such analysis.

Nevertheless, we also think that theoretical considerations should prevail over technical ones. We could afford to make this choice because mathematical analysis was not indispensable for reaching the aims of our study. That is to say, even when not demonstrating that the changes in interactive behaviour that our models predicted in response to varying stressor intensity, responsiveness and irritability actually satisfied formal criteria for abruptness, our three-dimensional graphs were sufficiently informative to give empiricists an idea of the kind of behavioural trajectories that they might expect to find in their data when seeing the mother–child dyad as a non-linear dynamic system. Accordingly, they were sufficient to reach our aims.

In general, we see our models as first attempts to model non-linear aspects of mother–child interaction that have a mainly heuristic function. At this stage, it seems premature to let model building attempts be constrained by the wish to construct models that can be analysed mathematically. Of course, this is not to deny that the possibility of mathematical analysis is an important *desideratum* when constructing future models that are aimed to explain then-available empirical data.

Finally, when writing our paper, we hoped that it would evoke a discussion about the possibilities of building dynamic systems models of mother–child interaction, the assumptions that should underlie such models, and the kind of behaviour that is generated by such models. Accordingly, we highly appreciate

that van der Maas and Raijmakers took the effort to clarify their criticisms to our approach by constructing a model of their own. We see the discussion that is emerging from their and our efforts to construct a non-linear dynamic systems model of mother-child interaction as an exciting development. It strengthens our conviction that applying dynamical systems methods can help us to identify the questions that should guide empirical research on the interaction process. When discussing which models are to be preferred, and when contrasting models to each other and to empirical data, we are opening new ways to really study developmental processes.

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